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The impact of cooling on CCD-based camera systems in the field of image luminance measuring devices

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Abstract
From general statements made about the cooling of the CCD sensor within image luminance measuring devices (ILMD), one tends to conclude that cooling has an important effect on the luminance measurement results. However, this general statement is not valid. Based on a detailed model for the properties and the operation of ILMDs, we present results of the measurements for parameters of cooled and non-cooled systems. The influence of the two different parameter sets on the results of luminance measurements is illustrated for variations in the signal level and the temperature conditions. In summary, it can be said that only for very special applications does the cooling of the CCD have a significant influence on the measurement uncertainty associated with the luminance measurement results. Normally, one can compensate for the little drawbacks of non-cooled systems by selecting the most appropriate measuring algorithms.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The effect of cooling on CCD-based camera systems is widely discussed in the literature [1]. Especially in the field of astronomical images using cameras with very long integration times in the range of minutes to hours with only one exposure per evaluated image, cooling is very important.

From general statements made about cooling, one tends to conclude that cooling has an important impact on the measurement results for image luminance measuring devices (ILMDs) [2]. However, this is not valid as a general statement.

Therefore, in section 2 of this paper a model for ILMDs is evaluated which is based mainly on their physical properties.

In section 3, model parameter estimations are presented for state-of-the-art interline transfer CCDs, the result of measurements made with both cooled and non-cooled Sony (ICX285AL) systems.

Finally, in section 4, the influence of the parameters on luminance measurement results and the associated measurement uncertainties are illustrated for different signal levels and temperature conditions.

2. Modelling of CCD-based ILMDs

Based on the work of Janesick [1, 3] and the current work in [4] and [5], a matrix of luminance values $\boldsymbol{L}$ that is measured with ILMDs can be modelled as follows:

$$\boldsymbol{L} = \frac{k_{\mathrm{L}}}{\bar{\boldsymbol{P}}_{\mathrm{L}}}\boldsymbol{E}'_{\mathrm{e}}; \qquad \boldsymbol{E}'_{\mathrm{e}} = \frac{\boldsymbol{S}-\bar{\boldsymbol{D}}}{k_{\mathrm{Sys}}\cdot t_{\mathrm{I}}\cdot A_{\mathrm{P}}\cdot \bar{\boldsymbol{P}}}. \tag{1}$$

The meanings of the symbols are as follows: k_{L} is the luminance calibration factor, $\boldsymbol{E}'_{\mathrm{e}}$ is the effective irradiance on the CCD and $\bar{\boldsymbol{P}}_{\mathrm{L}}$ is the lens shading.

The irradiance $\boldsymbol{E}'_{\mathrm{e}}$ is determined as the measured light signal $\boldsymbol{S}$ corrected for the measured dark signal $\bar{\boldsymbol{D}}$ and the quantities k_{Sys} the system transfer factor, t_{I} the integration time, A_{P} the pixel area and $\bar{\boldsymbol{P}}$ the CCD shading are taken into account. The measured signal $\boldsymbol{S} = \boldsymbol{S}_{\mathrm{E}} + \bar{\boldsymbol{D}}$ can be written as the sum of a light signal $\boldsymbol{S}_{\mathrm{E}}$ and a dark signal $\bar{\boldsymbol{D}}$:

$$\boldsymbol{S} = \underbrace{k_{\mathrm{Sys}}\cdot t_{\mathrm{I}}\cdot A_{\mathrm{P}}\cdot\bar{\boldsymbol{P}}\cdot\boldsymbol{E}'_{\mathrm{e}}}_{\boldsymbol{S}_{\mathrm{E}}\text{(light signal)}} + \underbrace{k_{\mathrm{Sys}}\cdot(\boldsymbol{Q}_{\mathrm{DS0}}+\boldsymbol{N}_{\mathrm{R}}+t_{\mathrm{I}}\cdot\boldsymbol{I}_{\mathrm{DS}}(T))+S_{\mathrm{off}}+\boldsymbol{N}_{\mathrm{Q}}}_{\bar{\boldsymbol{D}}\text{(dark signal)}}, \tag{2}$$

$$u^2(\boldsymbol{S}) = u^2(\boldsymbol{S}_{\mathrm{E}}) + u^2(\bar{\boldsymbol{D}}) + 2r(\boldsymbol{S}_{\mathrm{E}},\bar{\boldsymbol{D}})\cdot u(\boldsymbol{S}_{\mathrm{E}})\cdot u(\bar{\boldsymbol{D}}). \tag{3}$$

Table 1. Maximum luminance values (cd m^{-2}) for different integration times and binning modes (aperture value 4).

t_I	Standard resolution	Binning 2*2	Binning 4*4
100 µs	70 000	17 000	4200
15 s	0.46	0.11	0.028

The dark signal $\bar{\boldsymbol{D}}$ depending on $\boldsymbol{Q}_{\mathrm{DS0}}$ is the dark signal offset at the integration time $t_\mathrm{I} = 0$ s, $\boldsymbol{I}_{\mathrm{DS}}(T)$ is the dark signal generation rate (dark current, depending on the CCD temperature T and the integration time/integration mode), $\boldsymbol{N}_\mathrm{R}$ is the reset noise of the charge to voltage converter, $\boldsymbol{N}_\mathrm{Q}$ is the quantization noise of the analogue to digital converter (ADC) and S_{off} is the signal offset (bold values describe matrices as image data and values of the kind of $\bar{X}$ represent mean values from various images to reduce the temporal noise by signal averaging).

The uncertainty estimation in (3) is given with the correlation between the measured light and dark signals. Further evaluations given in this paper have been derived from [6], whereas the calculations have been carried out with Monte Carlo simulations.

Resulting from the Poisson distribution of the signal (counting photons) we can use the following model[1] function to calculate the signal noise depending on the signal level [3] with σ_0 the temporal dark signal noise and F the constant factor for consistency in the units:

$$\sigma^2(\boldsymbol{S}) = k_{\mathrm{Sys}} \cdot F \cdot \boldsymbol{S}_\mathrm{E} + \sigma_0^2. \tag{4}$$

This is the basic equation for the photon transfer method (PTM).

In standard photometric applications, integration times in the range of 100 µs to 15 s are applicable[2] (table 1).

Higher luminance values can be measured using an additional neutral density filter to reduce the value of the irradiance on the CCD.

2.1. Dark signal

The dark signal in (2) reads

$$\bar{\boldsymbol{D}} = k_{\mathrm{Sys}} \cdot (\boldsymbol{Q}_{\mathrm{DS0}} + \boldsymbol{N}_\mathrm{R} + t_\mathrm{I} \cdot \boldsymbol{I}_{\mathrm{DS}}(T)) + S_{\mathrm{off}} + \boldsymbol{N}_\mathrm{Q}. \tag{5}$$

The signal offset S_{off} is an ADC parameter and constant by correction using, for example, zero time integration. This means that the temperature dependence in the systems used for these tests has a negligible influence. The quantization noise $\boldsymbol{N}_\mathrm{Q}$ is small and does not depend on the ambient temperature. The temperature dependence of the reset noise $\boldsymbol{N}_\mathrm{R}$ is not so important, but the system clock and the setup of the correlated double sampling (CDS) are more important for the reset noise.

[1] The unit correction factor $F = \mathrm{e}^-$ is necessary to correct the mathematical relation $\mathrm{E}\{X\} = \mathrm{D}^2\{X\}$, for Poisson distributed random numbers. The physically correct unit is obtained.

[2] Additionally the so-called zero time integration $t_\mathrm{I} = 0$ s is available. This is a special mode for interline transfer CCDs to correct the influence of smear and to estimate the basic dark signal without using a mechanical shutter.

Besides the reset noise, the dark signal at zero integration time $\boldsymbol{Q}_{\mathrm{DS0}}$ influences the temporal dark signal noise σ_0. Therefore, we simplify (5) and use

$$\bar{\boldsymbol{D}} = t_\mathrm{I} \cdot \boldsymbol{I}_{\mathrm{DS,ADC}}(T) + \bar{\boldsymbol{S}}_0. \tag{6}$$

For the dark signal model a dark signal offset with $\bar{\boldsymbol{S}}_0 \sim \boldsymbol{N}(\bar{\boldsymbol{S}}_0, \sigma_0)$ is the dark signal at $t_\mathrm{I} = 0$ s integration time (including reset noise, quantization noise and the ADC offset effect itself and calculated during the camera calibration) and $\boldsymbol{I}_{\mathrm{DS,ADC}}(T)$ is the dark signal generation rate measured at the output of the ADC[3]. Finally, we have to investigate the temperature dependence of the parameters σ_0 and $\boldsymbol{I}_{\mathrm{DS,ADC}}(T)$.

It is known that the dark signal generation rate $\boldsymbol{I}_{\mathrm{DS}}(T, t_\mathrm{I})$ doubles every ~8 K increase in the CCD temperature [1], so we model

$$\boldsymbol{I}_{\mathrm{DS,ADC}}(T) = \boldsymbol{I}_{\mathrm{DS,ADC}}(T_0) \cdot \exp(\alpha_{\mathrm{IDS}} \cdot \Delta T_\mathrm{a}) \tag{7}$$

with α_{IDS} the temperature coefficient for the dark signal generation rate at the ADC level, T_a is the ambient temperature[4] and $\Delta T_\mathrm{a} = T_\mathrm{a} - T_0$ is the temperature difference between the ambient temperatures T_a during measurement and T_0 calibration (normally $T_0 = 25\,°\mathrm{C}$).

2.2. Light signal

For the light signal the system transfer factor k_{Sys} is the only temperature-dependent parameter. Due to electronic shutter systems the integration time is crystal controlled with a relative change of $<10^{-4}$ of the frequency (valid for the full temperature range of the crystal operational conditions). Thus, the temperature dependence is negligible[5].

$$\boldsymbol{S}_\mathrm{E} = k_{\mathrm{Sys}} \cdot (1 + \alpha_{\mathrm{kSys}} \cdot \Delta T_\mathrm{a}) \cdot t_\mathrm{I} \cdot A_\mathrm{P} \cdot \bar{\boldsymbol{P}} \cdot \boldsymbol{E}'_\mathrm{e}. \tag{8}$$

The measurement results show that the k_{Sys} value can be calculated independently of the luminance calibration factor k_L. However, in practice this is not necessary for the temperature coefficient α_{kSys} so we include the k_{Sys} value in the value of the luminance calibration factor k_L.

2.3. Luminance measurement

The luminance measurement function depends on the luminance calibration factor k_L and its relative temperature coefficient α_{kL} that describes the sensitivity change in the camera over the temperature range.

$$k_\mathrm{L} = k'_\mathrm{L} \cdot (1 + \alpha_{\mathrm{kL}} \cdot \Delta T_\mathrm{a}). \tag{9}$$

[3] The increase in the dark signal noise due to the photon shot noise resulting from the dark signal generation is negligible.

[4] For the practical analysis, all corrections were evaluated and applied using camera internal temperature sensors such as CCD temperature or the temperature of electronic components (ADC, etc).

[5] Attention: for systems with mechanical shutters the influence of the temperature-dependent integration time and its reproducibility is not negligible.

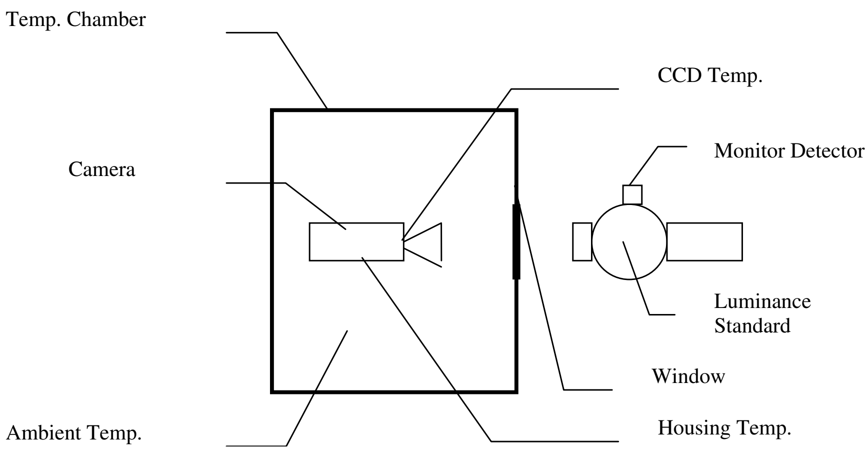

Figure 1. Measurement setup.

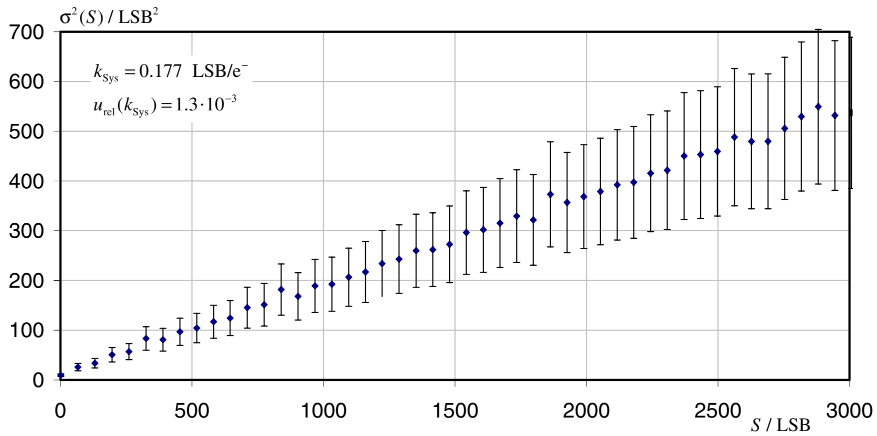

Figure 2. Example measurement to calculate the system transfer factor.[7]

2.4. Model summary

Summarizing the model equations, we get

$$\boldsymbol{L} = \frac{1}{A_P \cdot \overline{\boldsymbol{P}}\boldsymbol{P}_L} \cdot k_L \cdot \frac{\boldsymbol{S} - \bar{\boldsymbol{D}}}{k_{Sys} \cdot t_I} = k'_{Lc} \cdot \frac{\boldsymbol{S} - \bar{\boldsymbol{D}}}{t_I}, \tag{10}$$

$$\boldsymbol{L} = k_{Lc} \cdot \boldsymbol{S}_{eff}; \qquad k_{Lc} = k'_{Lc} \cdot (1 + \alpha_{kL} \cdot \Delta T_a),$$

$$\boldsymbol{S}_{eff} = \frac{\boldsymbol{S} - \boldsymbol{S}_0}{t_I} - \boldsymbol{I}_{DS,ADC}(T_0) \cdot \exp(\alpha_{IDS} \cdot (T_a - T_0)), \tag{11}$$

$$u^2_{rel}(\boldsymbol{L}) = u^2_{rel}(k_{Lc}) + u^2_{rel}(\boldsymbol{S}_{eff}), \tag{12}$$

$$u^2(k_{Lc}) = u^2(k'_{Lc}) \cdot (1 + \alpha_{kL} \cdot \Delta T_a)^2 + u^2(\alpha_{kL}) \cdot \Delta T_a^2 + u^2(\Delta T_a) \cdot (\alpha^2_{kL} + u^2(\alpha_{kL})), \tag{13}$$

$$u^2(\boldsymbol{S}_{eff}) = \frac{1}{N_{Pix}}(k_{Sys} \cdot F \cdot \boldsymbol{S}_E + \sigma_0^2) + t_I^2 \cdot u^2(\boldsymbol{I}_{DS,ADC}) + u^2(S_0). \tag{14}$$

For the calibration factor k'_{Lc} we combine the luminance calibration factor k_L and the influence of the pixel area A_P with the different shading images $\bar{\boldsymbol{P}}$ and $\bar{\boldsymbol{P}}_L$ with the system transfer factor k_{Sys} and its temperature dependence α_{kSys}. In this paper, we are interested only in the calibration value. Therefore, we use a scalar value here instead of the image or the matrix value which is necessary for the real system calibration. The value N_{Pix} represents the number of pixels or images for averaging during the measurement[6].

A luminance correction factor *lcf* is calculated, which allows one to compare the results of two different measurements:

$$lcf = \frac{\boldsymbol{L}_c(S = S_c, t_I = t_{I,c}, T_a = T_0)}{\boldsymbol{L}(S, t_I, T_a)}. \tag{15}$$

This correction factor has to be used if a measurement is made under conditions S, t_I, T_a while the system was calibrated for conditions S_c, $t_{I,c}$, T_0 (usually $S_c = 3500$ LSB, $t_{I,c} = 0.1$ s, $T_0 = 25\,°C$).[8]

3. Measurements

In the following section we describe the measurement setup and the parameter estimation from our measurement results.

[6] Attention: With this averaging we can reduce only the shot noise and the dark signal noise. However, we cannot reduce measurement uncertainties associated with other sources like calibration factor and the dark signal.

[7] The 'error bars' in the images representing the expanded $k = 2$ uncertainty of a 100-pixel area mean value.

[8] The digital signal is measured in LSB (least significant bit). Another common unit is DN (digital number).

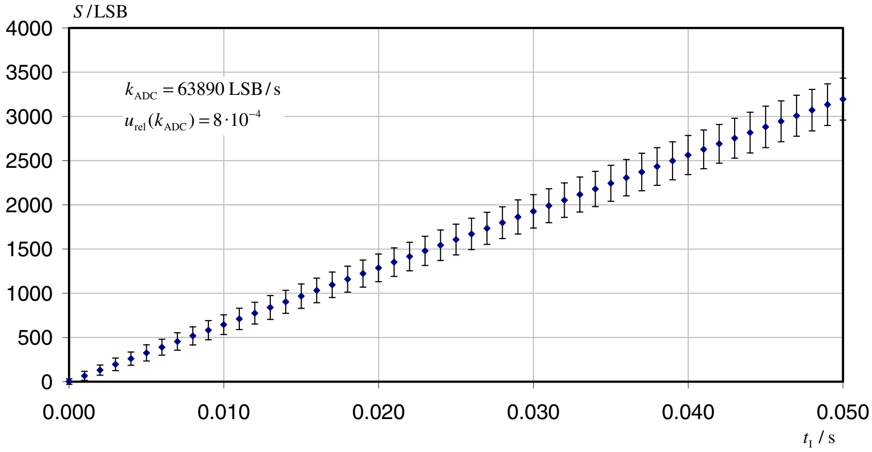

Figure 3. Example measurement to calculate the system sensitivity. (The 'error bars' represent 10 times the standard uncertainty of single pixel measurements.)

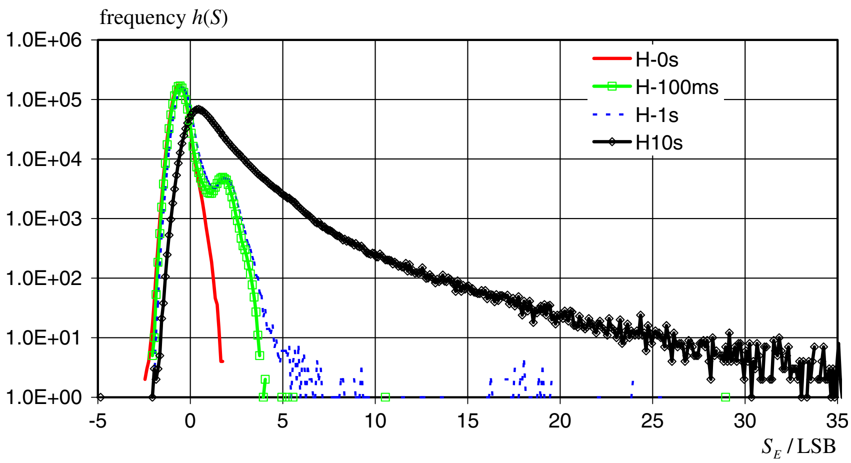

Figure 4. Non-cooled system.

At the end of this section, various measurement results are presented.

3.1. Measurement setup

A scientific CCD camera (12 bit quantization) was used for different investigations with measurement setups for dark and light signal measurements and was operated with a stabilized cooled system (CCD temperature is stabilized to $-15\,°\mathrm{C}$, referred to as the cooled system) and a not temperature-stabilized system, referred to as the non-cooled system.

Furthermore, we worked at a normal speed (25 MHz in the case of our DUT) and with a slow scan version (clock reduced by a factor of 8). All tests are made with and without a $V(\lambda)$ filter and with two different light sources.

1. *Luminance standard.* Standard illuminant A; supported with a monitor detector for the stabilization of the luminance value.
2. *White LED standard.* Temperature-stabilized white LEDs (conversion type); supported by a detector-based feedback system to keep the luminance value constant.

For this paper, we reduce the amount of data and present only the results obtained from the standard illuminant A measurements. The other measurement results with the LED standard are comparable.

Figure 1 shows the basic measurement setup. All dark measurements are carried out without a light source and a closed window of the temperature chamber.

3.2. Parameter estimation

First, the values of the model parameters with the associated uncertainties have to be estimated for different temperature setups. In a second step, we can calculate the temperature coefficients for these parameters. During the tests all measurements are made using regions with at least 1000 pixels.

We estimate the model parameter k_{Sys} and σ_0 by applying the well-known PTM and the generalized least-squares (GLS) regression method [7] to estimate the slope and intercept from (4).

A stable and monitored light source as a reference standard with a certified luminance value L_{Std} is used for the determination of the system calibration factor k_{ADC} (figures 2 and 3). The system calibration factor k_{Lc} for the end user[9] can

[9] The system calibration factor for the end user includes additionally the measurement uncertainty of the luminance standard.

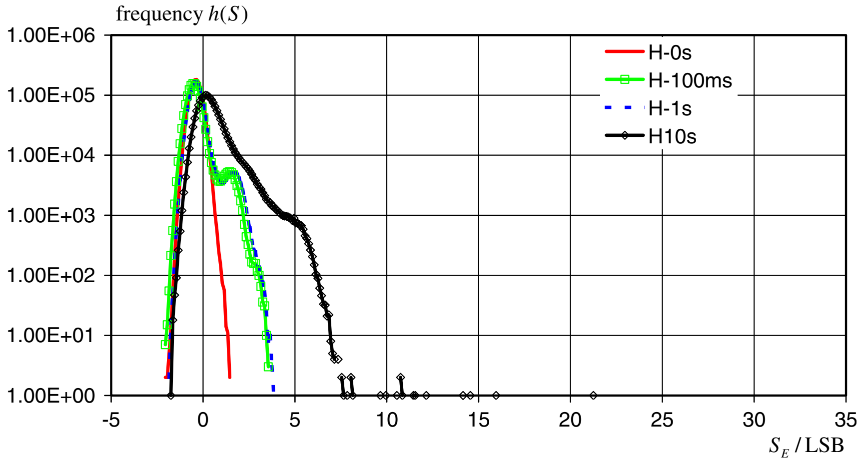

Figure 5. Cooled system.

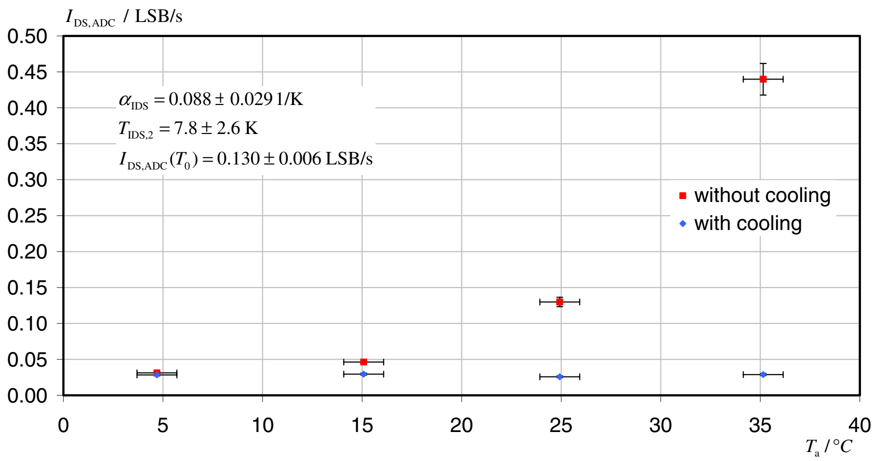

Figure 6. Measurement of the dark signal generation rate for a cooled system (blue diamonds) and a non-cooled system (red squares).

be calculated as follows:

$$k_{\mathrm{Lc}} = \frac{L_{\mathrm{Std}}}{k_{\mathrm{ADC}}}. \tag{16}$$

The calibration of the camera itself is not the topic of this paper; therefore, in the following sections we deal only with the parameter k_{ADC} and its uncertainty.

3.3. Measurement results

The dark signal measurements are presented first, and then we explain the light signal evaluations.

3.3.1. Dark signal measurements. The influence of the changes in the dark signal can be demonstrated by means of the faulty[10] pixel statistics. We call 'faulty pixels' all pixels with crystal traps or other defects in the crystal structure generating a higher dark current. Figures 4 and 5 show residual dark signal histograms of the S_{E} image for an ambient temperature of 20 °C of a 12 bit camera system and for integration times $t_{\mathrm{I}} = \{0\,\mathrm{s}, 100\,\mathrm{ms}, 1\,\mathrm{s}, 10\,\mathrm{s}\}$.

[10] Other names are hot pixels or blemish pixels.

Table 2. Basic noise estimation σ_0/LSB for different setups ($U(\sigma_0) = 0.2$ LSB).

	Normal speed 25 MHz	Slow scan 3.1 MHz
Cooled	2.8	1.9
Non-cooled	2.5	1.8

Table 3. Temperature coefficient $\alpha_{k\mathrm{ADC}}/\mathrm{K}^{-1}$ for different setups and expanded $k = 2$ uncertainty $U(\alpha_{k\mathrm{ADC}}) = 0.0002\,\mathrm{K}^{-1}$.

	With $V(\lambda)$ filter	Without $V(\lambda)$ filter
Cooled	−0.000 89	0.000 21
Non-cooled	−0.000 20	0.001 38

Figures 4 and 5 show that for integration times up to 1 s, the difference between a cooled and a non-cooled system is negligible. For an integration time of $t_{\mathrm{I}} = 10$ s, we have a significant number of pixels with a higher dark signal, and adapted algorithms have to be used for dynamic faulty corrections to compensate for this specific effect.

In a second step, the dark signal generation rate is calculated at the ADC level. In figure 6 the dark signal generation rate is low for the non-cooled system (red squares).

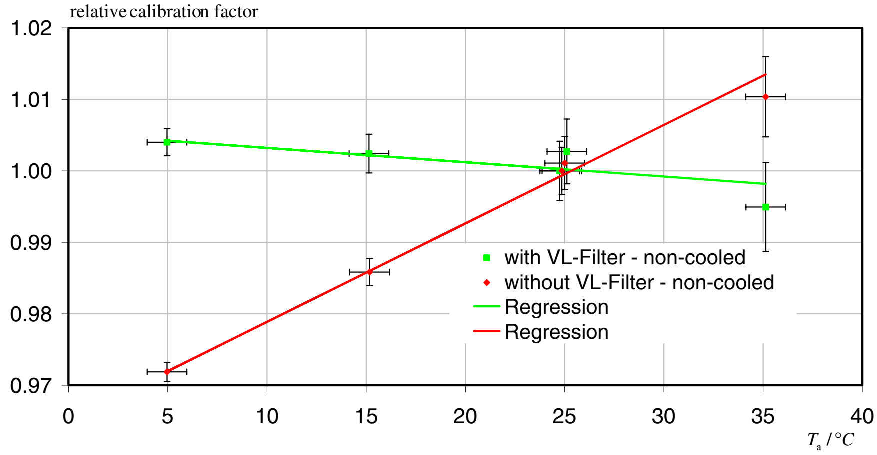

Figure 7. Relative calibration factor for a non-cooled camera with (green squares) and without (red diamonds) a $V(\lambda)$ filter.

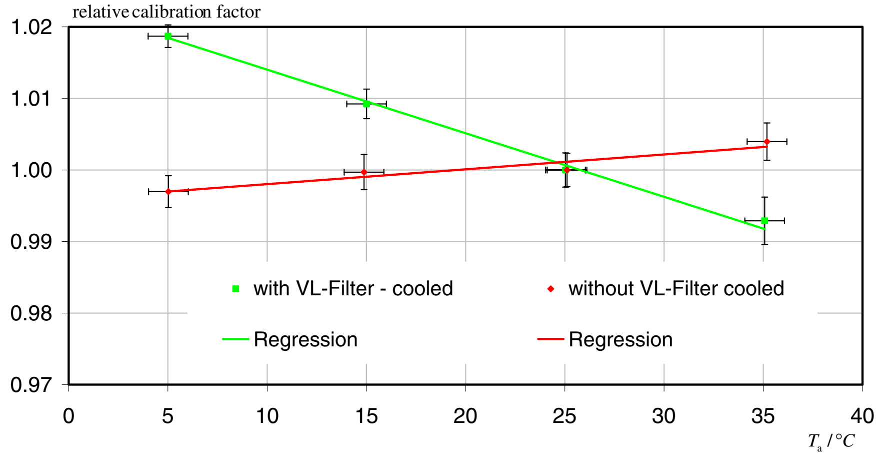

Figure 8. Relative calibration factor for a cooled camera with (green squares) and without (red diamonds) a $V(\lambda)$ filter.

The cooled and temperature-stabilized system (blue diamonds) has a constant dark signal generation rate because the dark signal depends on the stabilized CCD temperature ($T_{\mathrm{CCD}} = -15\,^\circ\mathrm{C}$). The basic noise estimation σ_0 (4) does not depend on the temperature.

The lower values in table 2 indicate that for this type of cameras, the slow scan mode reduces the dark signal noise[11].

3.3.2. Light signal measurements. The relative calibration factor (normalized to the calibration factor measured at $T_0 = 25\,^\circ\mathrm{C}$) for the cooled and the non-cooled systems with and without a $V(\lambda)$ filter was determined with a monitor-controlled luminance reference standard.

Summarizing the results in figures 7 and 8 we can calculate the temperature coefficients $\alpha_{k\mathrm{ADC}}$ for the different setups using a weighted least-squares linear regression [7]. Table 3 shows that the temperature coefficient of the $V(\lambda)$ filter is important. In our camera the $V(\lambda)$ filter was mounted on the front of the lens, so the temperature of the filter is approximately the ambient temperature. Usually the $V(\lambda)$ filter is either temperature-stabilized (extremely seldom) or is located between the CCD and the lens—and its temperature lies between the CCD temperature and the ambient temperature.

[11] Attention: not only is the dark signal itself responsible for the dark signal noise, all other electronic components in the signal chain can produce additional effects also. In our example, the reset noise from the current-to-voltage converter plays an important role.

4. Effects on measurement results and their uncertainties

The uncertainty evaluation depends on the model parameter estimated for the given system (cooled/non-cooled) under different ambient conditions and on the measurement procedure and algorithm used for capturing images.

In this section we show the necessary correction for the measured luminance values (including the corrections for the temperature dependence of the calibration factor and the influence of the dark signal parameters) based on the estimated model parameters for a calibration at $T_0 = 25\,^\circ\mathrm{C}$. Figure 9 shows the correction factor for the luminance values at the signal level $S = 3500$ LSB, for integration times $t_{\mathrm{I}} = 0.1$ s and for systems with the $V(\lambda)$ filter valid for areas of 100 pixels.

The uncertainty associated with the luminance correction is very low and thus negligible. It should be noted that the correction for the non-cooled (red diamonds) system is

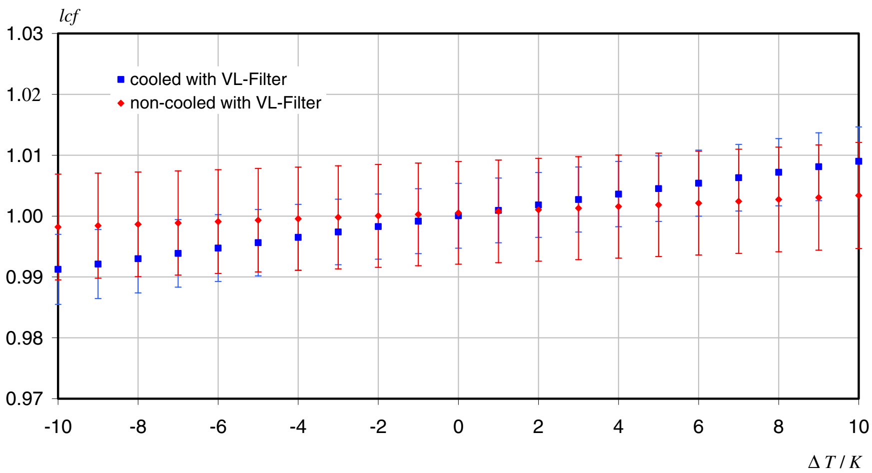

Figure 9. Luminance value correction factor over temperature and 'error bars' show the associated expanded $k = 2$ uncertainties for $S = 3500$ LSB and $t_I = 0.1$ s.

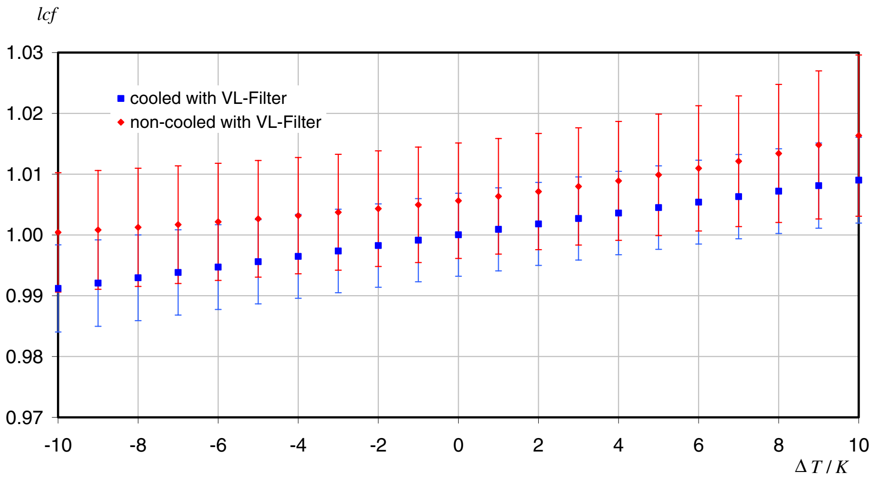

Figure 10. Luminance value correction factor over temperature and the associated expanded $k = 2$ uncertainty for $S = 350$ LSB and $t_I = 15$ s

lower than the correction for the cooled system (blue squares). Figure 10 shows the correction factor for luminance values at a 10 times lower luminance level and for a 150 times longer integration time.

For even lower signal levels (10% full load and a long integration time) in figure 10 the correction is low also, and some additional effects resulting from the dark current correction appear.

At the lowest signal level and for a long integration time, the correction for the non-cooled system is higher, and the uncertainty associated with this correction increases dramatically for high positive ambient temperature differences (figure 11).

From figure 12, we can see that the necessary correction due to temperature effects is very low and only significant for longer integration times and very low signal levels.

To reduce the effects caused by the temporal noise, one can use the mean of N images at one selected integration time t_I (multi image capture) or the mean over an image region. On the other hand, the high-dynamic image capture (using different images at different integration times and combining the results) can be used to reduce the influence of the dark current on dark areas of a luminance image with high contrast. This algorithm usually avoids the evaluation at low signal levels.

With these more or less time consuming methods, the user can reduce the measurement uncertainty associated with the luminance value [4] to his needs.

5. Summary

The influence of the cooling of the CCD and of the ambient temperature on the temporal dark signal noise is negligible. For the dark signal itself and the faulty pixels, the temperature-dependent effects can be corrected. The residual dark signal generation rate for non-cooled systems is <1 LSB/s for $T_a < 35\,°C$, which results in associated relative uncertainty distributions of less than 0.0004 for a 12 bit camera system at full scale. For non-cooled systems, one can correct the temperature-dependent effects.

Normally, one can compensate for the little drawbacks of non-cooled systems by selecting the most appropriate measurement algorithms. The use of cooled systems is recommended only for very long integration times ($t_I > 30$ s)

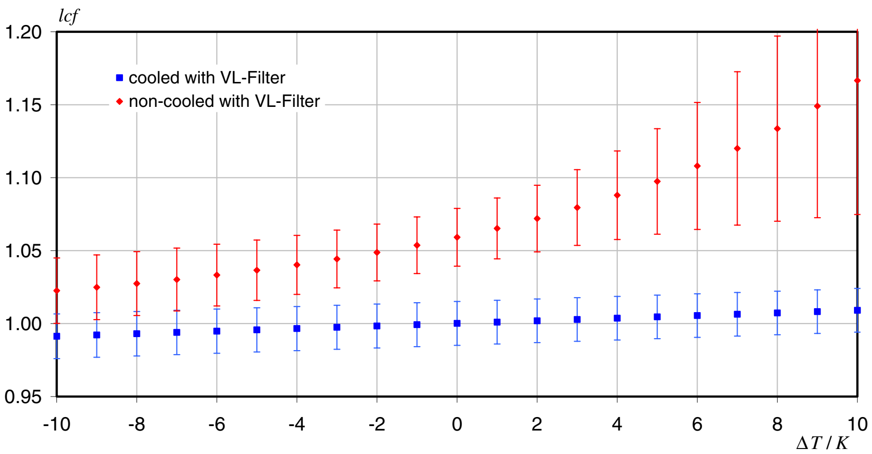

Figure 11. Luminance value correction factor over temperature and the associated expanded $k = 2$ uncertainty for $S = 35$ LSB and $t_I = 15$ s.

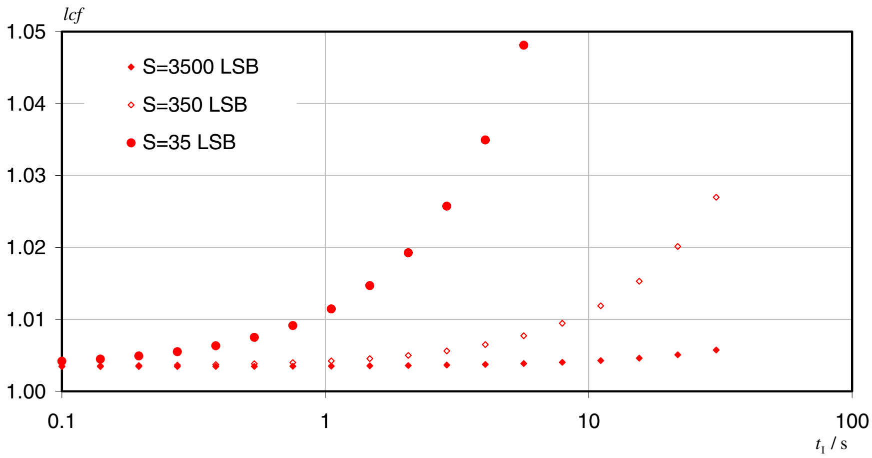

Figure 12. Luminance value correction factor for $\Delta T = +10$ K and for different signal levels over different integration times for a non-cooled system.

and for applications with restricted capture conditions (only single image capture available).

All statements can be applied to colour image measuring devices also.

Acknowledgment

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References

[1] Janesick J R 2001 *Scientific Charge-Coupled Devices* SPIE Press Monograph PM83 (Bellingham, WA: SPIE)

[2] Jenkins D R, Beuzekom D C, Kollman G, Wooley C B and Rykowski R 2001 Digital imaging colorimeter for fast measurement of chromaticity coordinate and luminance uniformity of displays *Proc. SPIE* **4295** 176–87

[3] Janesick J R 2007 *Photon Transfer: DN* $\rightarrow \lambda$ (Bellingham, WA: SPIE)

[4] Krüger U 2004 Ausgewählte Aspekte der ortsaufgelösten Erfassung stationärer Licht- und Farbverteilungen *Dissertation* ISBN 3-89959-167-4

[5] CIE 2008 *Characterisation of Imaging Luminance Measurement Devices (ILMDS)* Working Document TC2-59, Draft 1.0

[6] CIE 2008 *Determination of Measurement Uncertainties in Photometry* Working Document TC2-43, Draft 9

[7] Reed B C 1989 Linear least-squares fits with error in both coordinates *Am. J. Phys.* **57** 642–6